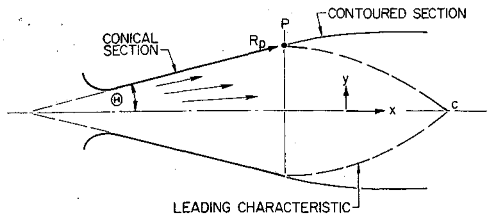


**Fig. 1 Nozzle geometry.**

where $c$ and $u$ are the local sound speed and velocity, respectively.

We are considering here the case of large Mach number so that $u$ is close to its thermodynamic limit and may be treated as constant. Therefore, from continuity the density is

$$\rho \simeq \rho_p(r_p/r)^2 \tag{4}$$

Where the subscript $p$ refers to the point where the leading characteristic starts, and $r$ is the radial distance from the origin. Assuming an isentropic flow, $p \propto \rho^\gamma$, of perfect gas, $p \propto \rho T \propto \rho c^2$, we have

$$c = c_p(r_p/r)^{\gamma-1} \tag{5}$$

and using the fact that $x \simeq r$

$$\mu \simeq (r_p/x)^{\gamma-1}/M_P \tag{6}$$

and $M_P = u/c_p$. Equation (3) becomes

$$dy/dx = (y/x) - (1/x)^{\gamma-1}/M_P \tag{7}$$

where the variables have been made nondimensional by dividing by $r_p$. At $x = 1$, $y = \sin\Theta \simeq \Theta$, where $\Theta$ is the nozzle half angle. The solution to Eq. (7) is

$$y = \Theta x + (1/x^{\gamma-2} - x)/M_P(\gamma - 1) \tag{8}$$

and the point where the leading characteristic crosses the axis is the point at which $y \to 0$, i.e., at

$$x_c = 1/[1 - M_P(\gamma - 1)\Theta]^{1/(\gamma-1)} \tag{9}$$

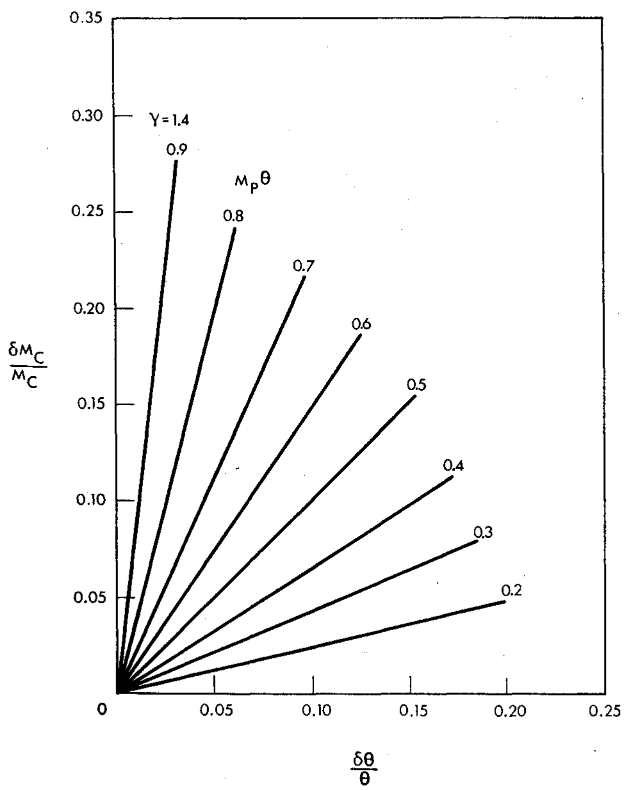


**Fig. 2 Variation in Mach number uncertainty vs angular uncertainty.**

The fractional error in $x_c$ caused by an error in the product $M_P\Theta$ will be

$$\delta x_c/x_c \simeq \delta(M_P\Theta)/[1 - M_P(\gamma - 1)\Theta] \tag{10}$$

Clearly if $M_P$ is large, a large error in $x_c$ can occur for small manufacturing errors in $\Theta$. The Mach number in the conical section varies as $1/\mu$ or

$$M_c = M_p(x_c)^{\gamma-1} \tag{11}$$

Assuming that $M_P$ is large Eqs. (10) and (11) yield

$$\delta M_c/M_c \simeq (\gamma - 1)M_P\delta\Theta/[1 - (\gamma - 1)M_P\Theta] \tag{12}$$

The error in $M_c$ is controlled by errors in the location of the junction between the conical-contoured sections and in the error in $\Theta$. For the data presented by Edenfield,[1] $\delta M_c/M_c \simeq 15\%$ with $\Theta = 12°$, $M_P \simeq 7$, $R_P \simeq 8$ in. This error could be caused if $\delta\Theta = 0.24°$. This error in $\Theta$ corresponds to an error in the diameter at the cone exit of about 0.333 in. which is somewhat large for usual machine tolerance.

The sensitivity of Mach number uncertainty to angular uncertainty is plotted in Fig. 2.

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# Experimental Investigation of a Luneberg Lens Antenna for Communications Satellites

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### Concept and Introduction

A LUNEBERG lens can be mounted at the end of a spin-stabilized synchronous satellite so that a small feed directed at the center of the lens can be rotated 360° around the lens. This feed is connected to the satellite receiver/transmitter by a coaxial cable and a rotating joint on the spin axis. The low-directive pattern of the feed is concentrated by the Luneberg lens to a narrow pencil beam. As the feed is counterrotated to the satellites spin motion by its own motor, this antenna beam can be locked on the earth or any specific place of it. Figure 1 is an outline drawing of the installed lens and feed arrangement.

Luneberg spheres are lenses that have a dielectric constant $\epsilon$ that obeys the following law:

$$\epsilon = n^2 = 2 - (r/a)^2$$

where $n$ = refractive index, $D/2 = a$ = outer radius of lens, and $r$ = radius to actual point; i.e., the dielectric constant in the center of the lens equals 2 and reduces to unity at the rim.

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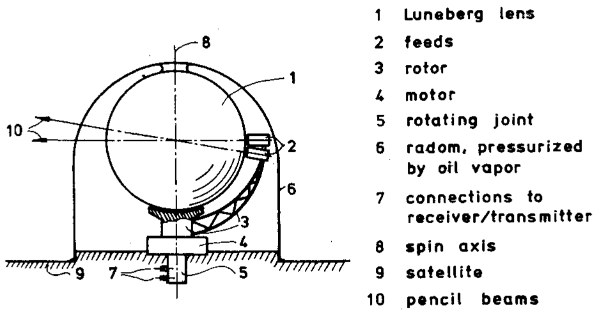


**Fig. 1 Luneberg lens antenna within satellite.**

Because it is difficult to achieve a smooth variation of the dielectric constant with radius, a stepwise approximation to the variation of $\epsilon$ with $r$ has been used by constructing the lenses of a number of concentric hemispherical shells each having a constant refractive index. Often adjustable density foams, such as polystyrene foam, are used. The number of shells varies from 10 to 24 or more. It is doubtful that the performance of a lens can be improved by increasing this number, because it is difficult to fabricate shells with a tolerance in $\epsilon$ less than $\pm 0.02$. Some years ago two 48-in.-diam Luneberg lenses were built, a 10-step and a 44-step lens. The 10-step design was somewhat superior to the other in many respects.[1]

### Measurements

To investigate this construction, four Luneberg lenses were tested in the indoor antenna range of the Deutsche Versuchsanstalt für Luft- und Raumfahrt (DVL) at downlink frequencies of 3.4 to 4.2 GHz. Short, square, open-end waveguide feeds were used. They showed a very similar *H*- and *E*-plane pattern (Fig. 2) and VSWR (voltage standing wave ratio) <2 from 3.5 GHz to 7.2 GHz; VSWR <1.14 from 3.7 GHz to 4.2 GHz. From their patterns the directivity gain was calculated with different methods.

Antenna diagrams of different 18-in.-diam Luneberg lenses both in receiving and transmitting mode were taken with this feed (Fig. 3). It was oriented to the lens in such a way, that maximum gain was obtained. In this case the open end of the feed usually was ~10 mm above the spheres skin; if it touched the skin, the gain was ~0.2 db less.

To test the homogeneity of the lenses, their feeds were allocated to maximum power gain position and only the lenses were rotated: the maximum difference of the signal

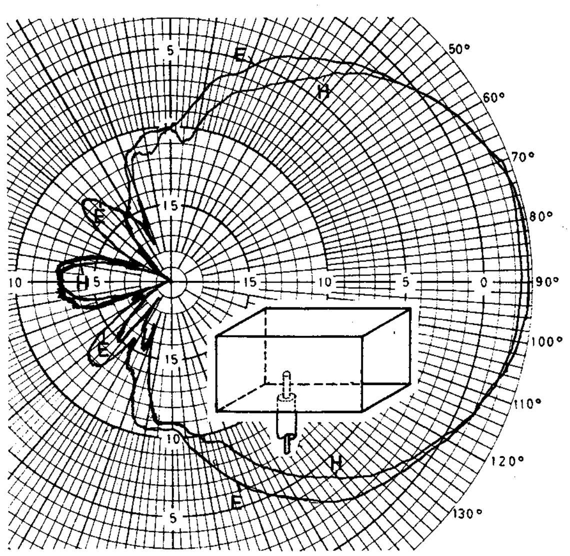


**Fig. 2 *H*- and *E*-plane antenna pattern of open-end waveguide feed at 4.0 GHz (power received in db).**

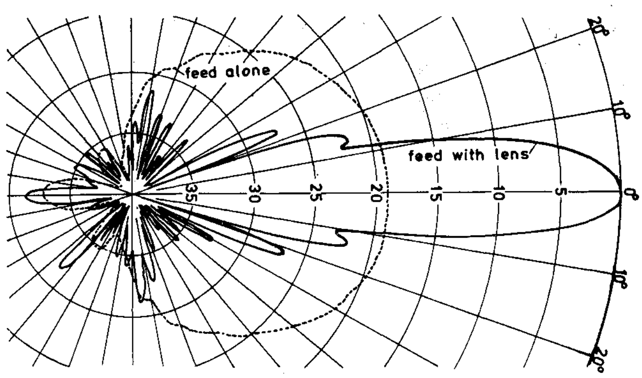


**Fig. 3 *H*-plane antenna patterns of feed and of feed with 17.3 in. $\phi$ Luneberg lens at 4.5 GHz.**

at the receiver was usually about 0.3 db at 4 GHz. An 18-in. lightweight lens showed 2 db maximum difference, but there were certain planes of each lens with a maximum difference of only 0.2 db and less. (It is possible to mount the lens to the spacecraft so that these planes are used.) The first sidelobes of the lenses were investigated in the same manner and the maximum difference was about 5 db.

Antenna patterns of the feed with the lens were taken by rotating the whole assembly. As the patterns in *E*- and *H*-plane of the feeds alone were quite similar, they were also similar with Luneberg lenses. At 4 GHz a half power beam width of ~10° was achieved with 18-in.-diam lenses; at 6 GHz this value was ~7° in both planes.

The first sidelobe level was usually between −20 db to −22 db in both planes. The backlobes were of the same order as those received from the feeds alone; the cross polarization showed the same behavior. It was −30 db and less—feed alone or with lens.

The following approach was used to determine the power gain of the antenna. The single feed was directed to the transmitting or receiving antenna in the test range. The signal on the receiver was noted. Now the Luneberg lens was placed in front of the feed and the signal on the receiver was reduced to the same value by means of a precision attenuator.

This attenuation was added to the calculated gain of the feed used. This sum represents the directivity gain from the feed plus the power gain from the Luneberg lens, but as the ohmic loss of the feed is very small, the sum can be taken as the power gain $G$ of this antenna configuration (all components of it were matched). The results are shown in Table 1. Nearly the same figures were achieved by comparison with a standard gain horn. To compare the results with figures evaluated at different frequencies and aperture diameters, a well-known formula[3] was used to evaluate the aperture efficiency

$$G = \eta(\pi D/\lambda)^2$$

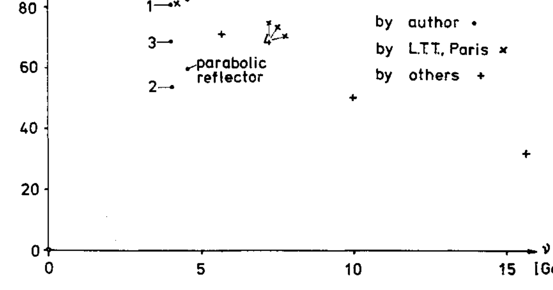


**Fig. 4 Aperture efficiency vs frequency of different Luneberg spheres of ~18 in. diam.**

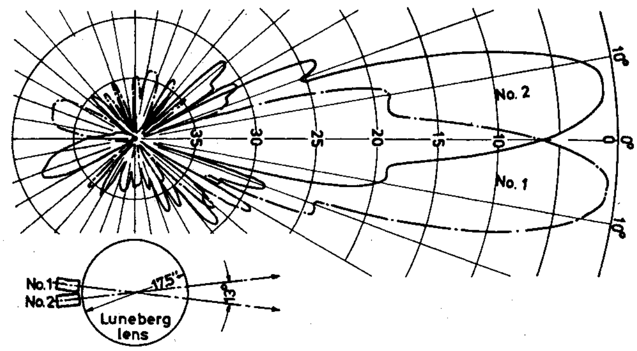


**Fig. 5 *H*-plane pattern of Luneberg lens with 2 feeds at 3.9 GHz.**

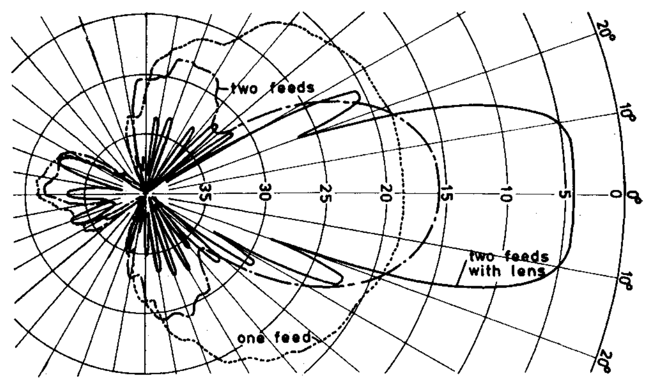


**Fig. 6 Luneberg lens with two feeds (assembly as shown in Fig. 5) parallel connected at 3.9 GHz.**

where $G$ = power gain, $\eta$ = aperture efficiency, $D$ = aperture diameter, and $\lambda$ = wavelength. The results are shown in Fig. 4.

Values of 0.8 and more for aperture efficiency obtained may be compared with the 0.5–0.6 value of an ordinary parabolic reflector antenna in the same range of $D/\lambda$ from 5–10. There is no aperture blocking from the feed and its support structure in the Luneberg lens configuration and there are less spillover losses. Nearly all energy propagated by the feed is collimated by the lens, which seems to be illuminated in an ideal manner.

A disadvantage of the lens antennas is their attenuation because of the absorption of the dielectric material. This absorption is proportional to the loss tangent ($\tan\delta$) and the $D/\lambda$ used. In recent years a polystyrene with very small impurities was produced. Its loss tangent could be held down to 0.00005 (at $\epsilon = 2.5$)[4] which is one order of magnitude less than any former value.

In connection with the fact that the frequencies used are only half of $X$-band radar applications, where until now much of the research work on Luneberg lenses was done, this could explain the unmeasurable small absorption loss and the high-aperture efficiencies of 0.8 and more.

A great advantage of Luneberg lenses is the possibility to use two or more feeds with the same sphere as antenna aperture without disturbing the resultant pencil beams. This is shown in Fig. 5, where two square open-end waveguide feeds (polarization parallel) were directed to the lens. Each feed has its own main lobe and there is no significant interference but the sidelobes between them are a little higher (−19 db) and the sidelobes at the outer sides are less (−24 db) than from one feed alone. The coupling between the two feeds touching each other at one side of their open ends was in the order of −20 db (compared with a precision attenuator); the VSWR remained low (e.g., without lens 1.15; with a light weight lens 1.05, and with a standard lens 1.40. It exhibits considerable variance at different points of the spheres skin, thus perhaps offering a sensitive method to test the homogeneity of the shells of the lens). By means of using special feeds, "beam-shaping" is possible (as shown in Fig. 6).

Because their mutual influence can be low, multiple feeds may be used onboard of a synchronous satellite with only one lens and so achieving multiple independent beams with constant attitude to each other. This may be helpful to solve frequency sharing and interference problems.

**Table 1 Power gain of tested Luneberg lenses at 4.0 GHz**

| No. | Luneberg-lens | Diameter $D$, in. | Weight, lb | Power gain $G$, db | Aperture efficiency, % |
|---|---|---|---|---|---|
| 1 | Emerson & Cuming Inc. | 17.5 | 35 | 24.5 | 81 |
| 2 | Emerson & Cuming Inc. | 17.5 | 7 | 22.7 | 54 |
| 3 | ELTRO | 20.7 | 53 | 25.2 | 68 |
| 4 | L.T.T.[a] | 17.3 | 40 | 24.6 | 85 |

[a] Lignes Telegraphiques et Telephoniques, Paris, France.

## Problems

As the weight of a Luneberg lens grows with the cube of its diameter, compared with the weight vs diameter squared relationship of a parabolic reflector, there are some weight saving problems in space applications. An 18-in.-diam Luneberg lens of pure polystyrene foam, which produces a 10° beam at 4 GHz, weighs 35 lb, but lightweight units of the same diameter are available with a weight of only 7 lb (the gain will be somewhat less because of higher absorption losses of the artificial dielectric material used) and it seems possible to fabricate them at only 2 to 3 lb. The matrix material still will be low-density polystyrene foam, which shows good mechanical properties to withstand accelerations during launch.

In recent years, several $C$-band Luneberg lenses were fabricated with excellent physical and electrical properties: 34-in. diam, 18-lb weight, isotropy better than 1 db, and gain factor >50%.[5]

Because of the absorption loss within the lens, there will be some temperature problems as the matrix material becomes soft at temperatures above 80°C (175°F). This is not important for the transmission power of the order of 10 w now used but will become a problem when this figure is increased by a factor of 10 and more. To meet temperature requirements connected with transfer orbits near sun, it is possible to fabricate Luneberg lenses of ceramic foam, thus increasing the temperature limit to some 1000°F and even more.[6]

Problems concerning radiation in space will be far less. From the standpoint of change in physical properties and hydrogen evolution during irradiation, polystyrene is one of the most stable of all polymers. It has been calculated from the radiation values of stationary orbits that the threshold will be reached after more than 50 yr and a 25% damage after ~2000 yr.

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